Chemical Engineering Journal 77 (2000) 37–41

Chemical Engineering Journal

www.elsevier.com/locate/cej

The use of simultaneous iterative reconstruction technique for electrical capacitance tomography ☆

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Abstract

High-quality reconstruction of capacitance tomography data is important to get the quantitative information from the cross sectional images of the multiphase pipe flow. Due to the complex nature of the capacitance sensors, the reconstruction algorithms well developed for medical tomography are not applicable. The main problems lie in two aspects. One is the 'soft-field' effect, the other is the limited number of measurements. To resolve these problems and get high-quality images, a reconstruction algorithm, named simultaneous iterative reconstruction technique (SIRT), often used in geology investigation, is introduced. According to the fastness of the process tomography and the smoothing effect of SIRT, some improvements have been made. These methods are compared to the well-known linear back projection algorithm (LBP) and the linear back projection thresholded algorithm widely used in capacitance tomography. A computer simulated and real eight-electrode capacitance tomography system has been used for the evaluation.

Keywords: Process tomography; Image processing; Reconstruction algorithm; SIRT

1. Introduction

In capacitance process tomography, the tomograms reconstructed are often distorted. The reasons are as follows. Firstly, the number of capacitance sensors mounted around the process to be imaged is often small in process tomography, and hence, the number of measurements used to reconstruct the cross sectional images is limited. Secondly, the capacitance sensors generate an inhomogeneous field and the sensitivity distribution inside the field depends on the parameter distribution. This kind of field is called 'soft field' compared to the 'hard field' in medical tomography. Due to these characteristics, the capacitance tomography data are very inaccurate and it is difficult to reconstruct high quality images.

In addressing these problems, several reconstruction methods have been developed and tested, such as linear back projection (LBP), filtered back projection (FLBP), model-based reconstruction (MOR), algebraic reconstruction technique (ART) and various iterative methods. For the characteristics of the capacitance sensor and the demand of the 'fast' process tomography, these methods have their own shortage either in precision, such as LBP, FLBP, or in speed, such as MOR, ART and iterative methods.

In this paper, a simultaneous iterative reconstruction technique (SIRT) often used in geology investigation is introduced. This method is based on the least square principle. It is insensitive to the errors of the measurement data. It can be used to reconstruct high quality images from the inaccurate data containing much noise. Furthermore, it is always convergent. Due to these advantages, SIRT is a good algorithm for the reconstruction of capacitance tomography.

However, SIRT also has its own disadvantages. First of all, the speed of reconstruction is slow. To reach a higher precision, it needs to iterate more times. It is unpractical for the on-line process tomography. But this method can be accelerated by some revision. It is found that after choosing an apposite weighting function, the iterative times can be reduced greatly.

The other disadvantage of SIRT is that it shows a smoothing effect which blurs the interface transition. Though this effect is much less than that of LBP, it cannot be neglected. To weaken the smoothing effect, an entropic thresholding method is introduced. Entropic thresholding is based on the maximum entropy principle. It has been shown that this thresholding method can be used to reduce the smoothing effect and improve the quality of the reconstructed tomograms.

☆ Supported by the National Natural Science Foundation of China.

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1385-8947/00/$ – see front matter

PII: S1385-8947(99)00134-5

This paper is organized in five sections. Section 2 presents the basic principle of SIRT and the improvements. Section 3 presents the experimental set-up used for algorithm performance evaluation. In Section 4, results are presented and discussed and finally, Section 5 gives the conclusion.

2. Simultaneous iterative reconstruction technique

In this section, the principle of SIRT and two improvements of SIRT according to the characteristics of capacitance tomography are presented.

2.1. The principle of SIRT

The system model for a capacitance tomography system can be simplified as:

$$C = S(X)X + E \tag{1}$$

where C is the vector of the measured capacitance, X, the vector of the dielectric constant distribution, S, the matrix of the sensitivity distribution inside the capacitance sensors, can be obtained previously using the finite-element method (FEM) in computer simulation. If the sensing domain is divided into some mesh grids, S means the variance of capacitance caused by the change of material in one pixel position. E represents the errors between the measured data and the estimated data. It is not practical to recalculate S during the process of tomographic imaging because calculating the sensitivity distribution is a very time-consuming task. Though, the sensitivity distribution depends on the dielectric constant distribution, the iterative technique can be used to eliminate this kind of non-linear property.

Correspondingly, the least square principle in capacitance tomography is to minimize the following function:

$$f(X) = (C - SX)^{\mathrm{T}}(C - SX) \tag{2}$$

By setting the derivative of Eq. (2) equal to zero, the prerequisite satisfying the least square principle is:

$$S^{\mathrm{T}}C = S^{\mathrm{T}}SX \tag{3}$$

To get the solution of Eq. (3), the following iterative algorithm is used:

$$X^{(0)} = S^{\mathrm{T}}C$$

$$X^{(k+1)} = X^{(k)} + \lambda^{(k)}(S^{\mathrm{T}}C - S^{\mathrm{T}}SX^{(k)}) \tag{4}$$

This iterative algorithm, what is called, the simultaneous iterative reconstruction technique.

Here, the dielectric constant distribution X, can be represented by the gray level distribution G; the measured capacitance C, can be replaced by the normalized capacitance N. Then, the expression of the SIRT is described as follows:

$$G^{(k+1)} = G^{(K)} + \frac{1}{\sum_i S_i}\sum_i (N_{m,i} - N_{c,i}^{(k)})S_i \tag{5}$$

where the 'i' subscript indicates the different capacitance measurements, k is the iteration number, $N_{m,i}$, the measured capacitance, $N_{c,i}$, the simulated capacitance which is calculated by the simplified model mentioned in Eq. (1).

From the principle of the SIRT, it can be seen that the gray level distribution is revised according to the sum of all the errors between the measured and the simulated data. This is why the SIRT can inhibit the noise in the measured data in effect.

2.2. Two improvements

As described in the introduction section, the SIRT has two main disadvantages when used in on-line process tomography. One is the slow speed, the other is the smoothing effect. To address these two problems and improve the quality of the SIRT, two methods have been applied and tested, which are described as follows.

2.2.1. The choice of weighting function

As we know, in iterative method, the pace of each iteration plays an important role on the convergent speed. By choosing an apposite weighting function which changes the iterative pace, the convergent speed of the SIRT can be accelerated. The weighting function chosen in this work is as follows:

$$\lambda^{(k)} = \alpha + \frac{\beta}{k} \tag{6}$$

where k is the iteration number, α and β are relaxation factors.

The expression of the SIRT added the weighting function is:

$$G^{(k+1)} = G^{(K)} + \left(\alpha + \frac{\beta}{k}\right)\frac{1}{\sum_i S_i}\sum_i (N_{m,i} - N_{c,i}^{(k)})S_i \tag{7}$$

The experiment experiences show $\alpha = 1.5$, $\beta = 2.0$. Such values ensure the convergence of the SIRT and have proved capable of improving the convergent speed for all the regimes tested in this work. It is also found that different types of regimes have their own best values of α and β, which can accelerate the convergent speed faster. So, if the information of the type of the measured regimes could be considered into the weighting function, the speed of the SIRT would be improved further.

2.2.2. Entropic thresholding method

Due to a little smoothing effect of the SIRT, an entropic thresholding method is used. The entropic thresholding method is based on the maximum entropy principle which has proved to be very applicable in the situation where the measured data are limited. The total entropy of the cross sectional image is defined as:

$$\mathrm{HT}_s = -\sum_{i=1}^{s}\frac{p_i}{P_s}\ln\left(\frac{p_i}{P_s}\right) - \sum_{i=s+1}^{n}\frac{p_i}{1-P_s}\ln\left(\frac{p_i}{1-P_s}\right) \tag{8}$$

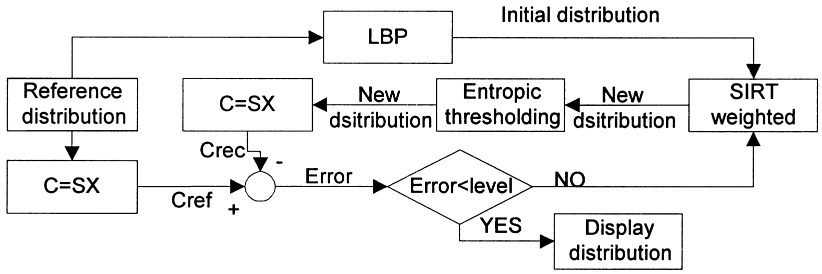

Fig. 1. A schematic diagram for the SIRT weighted and thresholded.

where

$$P_s = \sum_{i=1}^{s} p_i \tag{9}$$

where p_i is the probability of the occurrence of the ith gray level, s represents the thresholding gray level which classified the tomogram into object and background.

The value of s which maximizes HT_s is the thresholding gray level.

For the sake of analysis, we name the SIRT with these two improvements as the SIRT weighted and thresholded.

3. Evaluation experiment

For evaluating the SIRT and the performance of the improvements, a computer simulated and real eight-electrode capacitance sensor system based on FEM has been used with different tested dielectric constant distribution as references. The tested references and the evaluation criteria are outlined in this section.

3.1. Data set used

In this work, two-component distributions are considered and different cases of stratified, slug and bubble flow regimes are simulated. The number of mesh elements used to reconstruct these cases is 841. The measurement circuit is based on AC-excitation.

3.2. Evaluation criteria

Both qualitative and quantitative evaluation methods are used in this work. Qualitative evaluation involves visual comparisons of the different reconstruction methods including LBP thresholded, SIRT weighted, SIRT weighted and thresholded. Quantitative evaluation involves calculation of the normalized mean square error, the normalized mean absolute error, the correlation coefficient and the reconstruction speed. The normalized mean square error is sensitive to big errors of a few elements, while the normalized mean absolute is sensitive to small errors of many elements. The correlation coefficient indicates the spatial similarity between the test reference and reconstructed distribution. Quantitative evaluation criteria are defined as follows.

3.2.1. Normalized mean square error

The normalized mean square error is defined as:

$$\delta d = \left[\frac{\sum_{i=1}^{N} \left(g_i^{\mathrm{ref}} - g_i^{\mathrm{rec}} \right)^2}{\sum_{i=1}^{N} \left(g_i^{\mathrm{ref}} - \bar{g}^{\mathrm{ref}} \right)^2} \right]^{1/2} \tag{10}$$

where g_i^{ref} is the test reference gray level for pixel element i, g_i^{rec}, the reconstructed gray level of the pixel element i,

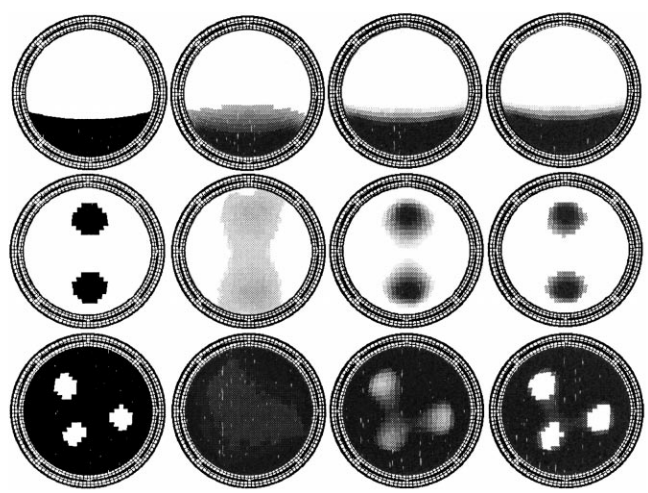

Fig. 2. Visual evaluation of the reconstructed tomograms when using LBP threholded, SIRT weighted, SIRT weighted and thresholded relative to the test reference for synthetic stratified, slug and bubble flows.

Table 1
Imaging of a stratified distribution

	δd	δr	R_{xy}	Iterative times
LBP	0.404443	0.349336	0.919878	1
LBP thresholded	0.371682	0.239853	0.930831	1
SIRT	0.223822	0.107903	0.975067	53
SIRT weighted	0.224357	0.111262	0.975012	22
SIRT weighted and thresholded	0.224329	0.110677	0.975015	22

Table 2
Imaging of a slug distribution

	δd	δr	R_{xy}	Iterative times
LBP	0.868578	1.498516	0.553796	1
LBP thresholded	0.859006	1.327148	0.560509	1
SIRT	0.443963	0.534733	0.903063	82
SIRT weighted	0.438385	0.520694	0.905382	50
SIRT weighted and thresholded	0.381719	0.339111	0.936994	50

Table 3
Imaging of a bubble distribution

	δd	δr	R_{xy}	Iterative times
LBP	0.940699	0.128569	0.382870	1
LBP thresholded	0.940669	0.128569	0.382870	1
SIRT	0.605353	0.068845	0.815606	313
SIRT weighted	0.604840	0.068755	0.815958	201
SIRT weighted and thresholded	0.632182	0.048769	0.840851	201

$\bar{g}^{\text{ref}}$, the average gray level. The best algorithm will have the minimum value of normalized mean square error.

3.2.2. Normalized mean absolute error

The normalized mean absolute error is defined as:

$$\delta r = \frac{\sum_{i=1}^{N} \left| g_i^{\text{ref}} - g_i^{\text{rec}} \right|}{\sum_{i=1}^{N} \left| g_i^{\text{ref}} \right|} \tag{11}$$

The best algorithm will have minimum value of normalized mean absolute error.

3.2.3. Correlation coefficient

The correlation coefficient is defined as:

$$R_{xy} = \frac{\sum_{i=1}^{N} \left(g_i^{\text{rec}} - \bar{g}^{\text{rec}}\right)\left(g_i^{\text{ref}} - \bar{g}^{\text{ref}}\right)}{\left[\sum_{i=1}^{N} \left(g_i^{\text{rec}} - \bar{g}^{\text{rec}}\right)^2 \sum_{i=1}^{N} \left(g_i^{\text{ref}} - \bar{g}^{\text{ref}}\right)^2\right]^{1/2}} \tag{12}$$

where $\bar{g}^{\text{rec}}$ is the average gray level. The best algorithm will have the maximum value of correlation coefficient.

3.3. Reconstruction speed

The relative additional computational cost over LBP algorithm reconstruction time is calculated. The iteration times of the SIRT and the SIRT with the improvements are calculated and used for comparison.

3.4. Implementation

The reconstruction method described in this paper can be achieved by using a scheme as illustrated in Fig. 1.

4. Results and discussion

In this section, the simulation and real experiment results will be presented and discussed.

In simulation experiments, samples of stratified, slug and bubble flow tomograms reconstructed by LBP thresholded, SIRT weighted, SIRT weighted and thresholded are visually

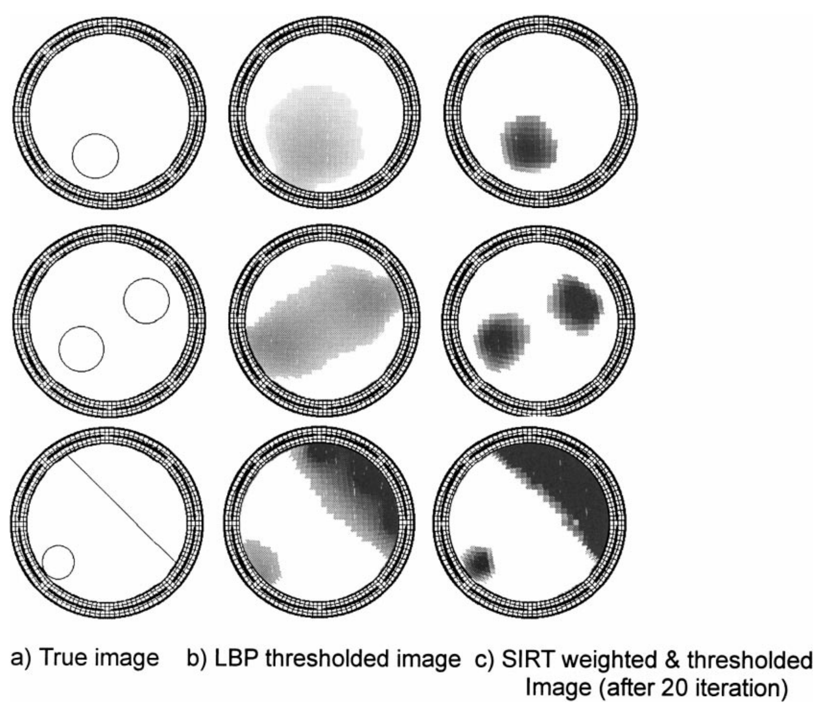

Fig. 3. Visual comparison of the reconstructed tomograms in real experiments.

compared in Fig. 2. The iteration of all the SIRT methods stops at the moment when the standard error between the reference measured and simulated capacitance decreases to 1% of the initial error.

From these comparisons, it can be seen that (a) Using the SIRT in reconstruction of capacitance tomography, data can generate high quality tomograms in spite of a little smoothing effects. (b) The entropic thresholding method can reduce the smoothing effects and improve the quality of the tomograms further.

Quantitative evaluation of the results shown in Fig. 2 are listed in Table 1 for the stratified case, Table 2 for the slug case and Table 3 for the bubble case. The time cost of the LBP methods is about one iterative time of the SIRT.

From these tables, it can be seen that (a) The values of the quantitative evaluation criteria (δd, δr and R_{xy}) of the SIRT methods are much better than that of the LBP thresholded methods. (b) The weighting function used in this work obviously accelerates the convergent speed of the SIRT. (c) The entropic thresholding method also improves the quality of the tomograms.

In real experiments, the results of four tests are presented. In these tests, the flow regimes are simulated by placing polypropylene bars or beads in the capacitance sensor. The final image obtained by the SIRT weighted and thresholded method after 20 iteration is given in Fig. 3, alongside the LBP thresholded image and the true image, for comparison.

5. Conclusions

In this paper, it has been shown that the SIRT can be used to significantly enhance the reconstructed image for a capacitance tomography system. The two improvements, the weighting function and the entropic thresholding method, do eliminate the disadvantages of the SIRT to some extent. It has also been pointed out in Section 2 that if some prior information of the dielectric distribution could be added, the effect of the improvements would be better.

The experiments are based on a computer simulated and real eight-electrode capacitance tomography system.

6. Further Reading

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